647510 **BSCalgCC5010** **Seat No :________**

B.Sc. Semester - 5 (Remedial) (*CBCS*) Examination
March/April-2022(Old Course)
MATHEMATICAL ANALYSIS-1 AND ABSTRACT ALGEBRA-1 (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(A) **Attempt any one.** (07)

1.Prove that any subset of discrete metric space is both open and closed.

2.Prove that the closure of a subset of a metric space is closed set.

Q.1(B) **Attempt any one.** (04)

1. In usual notations prove that (R,d) is a metric space.
2. Prove that every finite subset of any metric space is closed.

Q.1(C) **Attempt any one.** (03)

1.Fillup the following blanks.

(i) If E = [1,3] is a subset of metric space R then E^o = ________.

(ii) Let (X,d) be a metric space & $E \subset X$. Then set E is said to be dense if______.

(iii) If (X,d) is a discrete metric space and $a \in X$ then $N(a,\delta)$ = _________.

2. If $A = \{m+\frac{1}{n} / m,n \in N\} \subset R$ .Then find A'.

Q.2(A) **Attempt any one.** (07)

1. Let f be bounded function defined on [a,b]. P and P* are two partitions of [a,b] such that $P\subset P^*$. Then prove that $L(P,f)\leq L(P^*,f))\leq U(P^*,f))\leq U(P,f)$.
2. State and prove general form of first mean value theorem of integral Calculus.

Q.2(B) **Attempt any one.** (04)

1. If function f is defined as ;

f(x)=0, if x is rational

=1, if x is irrational.

Then show that f is not R-integrable on any interval.

2.Show that $-\pi^5 \leq \int_0^{\pi} x^4(3sinx + 4cosx)dx \leq \pi^5$.

Q.2(C) **Attempt any one.** (03)

1. If f(x) = 20/x , $x \in [2,20]$, partition P={2,,4,5,20} then find ‖ P ‖.
2. State Darboux's theorem.

Q.3(A) **Attempt any one.** (07)

1. State and prove second mean value theorem of integral calculus.
2. Show that G is a commutative group if $(ab)^i = a^i b^i$ for $\forall$ a,b $\in$ G, for any three consecutive integers i.

Q.3(B) **Attempt any one.** (04)

1. Let $f(x) = [x]$, $x \in [0,3]$. Show that $f \in R_{[0,3]}$. Where $[x]$ denotes the greatest among all integers not greater than x.
2. Prove that G is a group, where $G = \{a + b\sqrt{3} \;/\; a,b \in Q,\ a^2+b^2 \neq 0\}$ under multiplication of real numbers.

Q.3(C) **Attempt any one.** (03)

1. In usual notations state the formula of fundamental theorem of integral calculus.
2. If a is the only element of a group G having order n>1. Then show that n=2.

Q.4(A) **Attempt any one** (07)

1. Prove that a finite non- empty subset H of a group G is a subgroup of G,if $ab \in H$, $\forall\ a,b \in H$.
2. State & prove Lagrange's theorem for a subgroup of a finite group G.

Q.4(B) **Attempt any one.** (04)

1. If G is a finite group, then prove that $o(a)/o(G)$,$\forall\ a \in G$.
2. If $H \leq G$ then show that $x^{-1}Hx = \{ x^{-1}hx \;/\; h \in H\}$ is a subgroup of G, $\forall\ x \in G$.

Q.4(C) **Attempt any one.** (03)

1.Define even and odd permutation.

2. Find o(f) where $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 2 & 6 \end{pmatrix} \in S_6$.

Q.5(A) **Attempt any one.** (07)

1. If $\emptyset : G \rightarrow G'$ is an isomorphism of groups, then prove that $\emptyset^{-1} : G' \rightarrow G$ is also an isomorphism.
2. Prove that relation 'isomorphism of groups' is an equivalance relation.

Q.5(B) **Attempt any one** (04)

1. Show that $(R^+ , \cdot) \cong (R , +)$.
2. Let G be a group & let $g \in G$ be a fixed element of G. Then prove that the mapping $i_g : G \rightarrow G$, $i_g(x) = gxg^{-1}$ is an isomorphism, $\forall\ x \in G$.

Q.5 (C) **Attempt any one.** (03)

1. Find order of cyclic subgroup generated by element 2 of Z_8.
2. If $\mu = (1\ \ 3\ \ 4)(2\ \ 5) \in S_6$, then find μ^{-1}.

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